Conics review sheet:
a) A ball is thrown from a height of 7 feet at a 30° angle to the surface of the earth at 80 ft per second.

Write parametric equations for its displacement. Find its maximum height.
At what time does it hit the ground? How far does it go (horizontally)?
How far away is it from its starting point?
Find the latus rectum, the vertex and the focus

1. Graph the conic with equation $5x^2 + 20x - 12y^2 + 24y - 52 = 0$ with attention to focus directrix, asymptotes, etc.

2. Find an equation for the conic with directrix y=–3 focus at (0,3) and e = ½. Graph it with attention to the usual features.

3. Find the equation of the conic with latus rectum = 8, e = 1 and vertex at the origin

4. Accurately graph these conics with attention to the usual (including rotation angle)

a) $6x^2 + 5xy - 6y^2 = 78$

b) $2x^2 + \sqrt{3}xy + y^2 - 12 = 0$

c) $x^2 + 6xy + 9y^2 + \sqrt{10}x - 18 = 0$

5. Accurately graph these conics which attention to the usual (starting with eccentricity)

$a)\ r = \dfrac{30}{2 - 8\cos\theta}$

$b)\ r = \dfrac{30}{8 + 2\sin\theta}$

$c)\ r = \dfrac{10}{3 + 3\cos\theta}$

6. Convert to rectangular form:

$a)\ \begin{matrix} x = 3\cos t \\ y = -5\sin t \end{matrix}$ $b)\ \begin{matrix} x = \cos 2t \\ y = \sin t \end{matrix}$ $c)\ \begin{matrix} x = e^{3t} \\ y = e^{-5t} \end{matrix}$ $d)\ r = \tan\theta$

7. A searchlight mirror in the form of a parabola of revolution has focal length of 3'. Its maximum diameter is 4'. What is the depth of the mirror.

Be able to use combinations of focus, directrix, center eccentricity, asymptotes to write equation of conics and to graph them. Review your three quizzes

Conics review sheet: ANSWERS

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$$x = 40\sqrt{3}t$$
$$y = 7 + 40t - 16t^2$$

$$y - 7 = -16\left(t - \frac{5}{4}\right)^2 + 25$$

$$y\max = 32 \ \ at \ \ t = \frac{5}{4} \ \ and \ \ x = 50\sqrt{3}$$

$$y = 0 \ \ at \ \ t = \frac{40 + \sqrt{1600 + 4 \bullet 7 \bullet 16}}{32} = \frac{40 + 8\sqrt{25 + 7}}{32} = \frac{40 + 32\sqrt{2}}{32} = \frac{5 + 4\sqrt{2}}{4}$$

$$x = 40\sqrt{3}\frac{5 + 4\sqrt{2}}{4} = 10\sqrt{3}\left(5 + 4\sqrt{2}\right)$$

$$vertex\left(50\sqrt{3}, 32\right), \ t = \frac{x}{40\sqrt{3}} \Rightarrow y = -16\left(\frac{x}{40\sqrt{3}}\right)^2 + ... \Rightarrow 300y = x^2 + ...$$

$$latus \ \ rectum = 300, \ focus = 32 - 75 = \left(50\sqrt{3}, -43\right)$$

1. Graph the conic with equation $5x^2 + 20x - 12y^2 + 24y - 52 = 0$ with attention to focus directrix, asymptotes, etc.

$$5x^2 + 20x - 12y^2 + 24y = 0$$
$$5(x + 2)^2 - 12(y - 1)^2 = 20 - 12 + 52 = 60$$
$$\left(\frac{x + 2}{\sqrt{12}}\right)^2 - \left(\frac{y - 1}{\sqrt{5}}\right)^2 = 1$$

$$center = (-2, 1) c = \sqrt{17}, \ e = \sqrt{\frac{17}{12}}$$

$$as: \ (y - 1) = \pm\sqrt{\frac{5}{12}}(x + 2)$$

$$dir: \ e = \sqrt{\frac{17}{12}} = \frac{\sqrt{17} - \sqrt{12}}{\sqrt{12} - d} \Rightarrow d = \frac{12}{\sqrt{17}} \Rightarrow x = -2 \pm \frac{12}{\sqrt{17}}$$

2. Find an equation for the conic with directrix y=−3 focus at (0,3) and e = ½. Graph it with attention to the usual features. V=(0,1) and (0,9), center=(0,5)

$$B^2 = 4^2 - 2^2 = 12 \Rightarrow B = \sqrt{12}$$
$$\Rightarrow \left(\frac{x}{\sqrt{12}}\right)^2 + \left(\frac{y - 5}{4}\right)^2 = 1$$

3. Find the equation of the conic with latus rectum = 8, e = 1 and vertex at the origin

$$y^2 = 8x \Rightarrow r^2\sin^2\theta = 8r\cos\theta \Rightarrow r^2\sin^2(\theta - \alpha) = 8r\cos(\theta - \alpha)$$

4. Accurately graph these conics with attention to the usual (including rotation angle)

$$6x^2+5xy-6y^2=78 \quad B^2-4AC>0 \ \textit{hyperbola}$$

$$\cot 2\theta=\frac{12}{5}\Rightarrow \cos 2\theta=\frac{12}{13}\Rightarrow \cos\theta=\sqrt{\frac{1+\frac{12}{13}}{2}}=\frac{5}{\sqrt{26}},\ \sin\theta=\frac{1}{\sqrt{26}}$$

a) $\frac{1}{26}\left[6(5x'-y')^2+5(5x'-y')(x'+5y')-6(x'+5y')^2\right]=78$

$$\frac{1}{26}\left[x'^2(150+25-6)+x'y'(-60+125-5-60)+y'^2(6-25-150)\right]=78$$

$$\frac{1}{26}\left(169x'^2-169y'^2\right)=\frac{13}{2}\left(x'^2-y'^2\right)=78\Rightarrow\frac{x'^2}{12}-\frac{y'^2}{12}=1$$

asymptotes at right angles, center origin, $A=\sqrt{12},C=\sqrt{24}$

$$2x^2+\sqrt{3}xy+y^2-12=0 \qquad B^2-4AC=3-8=-5 \qquad \textit{ellipse}$$

$$\cot 2\theta=\frac{1}{\sqrt{3}}\Rightarrow 2\theta=60^\circ\Rightarrow\theta=30^\circ$$

b) $\frac{1}{4}\left[2\left(x'\sqrt{3}-y'\right)^2+\sqrt{3}\left(x'\sqrt{3}-y'\right)\left(x'+y'\sqrt{3}\right)+\left(x'+y'\sqrt{3}\right)^2\right]=12$

$$\frac{1}{4}\left[x'^2(6+3+1)+x'y'\left(-4\sqrt{3}+3\sqrt{3}-\sqrt{3}+2\sqrt{3}+y'^2(2-3+3)\right)\right]=12$$

$$\frac{1}{48}\left(10x'^2+2y'^2\right)=1\Rightarrow\frac{x'^2}{4.8}+\frac{y'^2}{24}=1,\ \textit{focus at } \sqrt{19.2}=4\sqrt{1.2}=\left(0,\pm8\sqrt{.3},\right)$$

$$x^2+6xy+9y^2-18+\sqrt{10}x=0 \qquad B^2-4AC=0 \quad \textit{parabola}$$

$$\cot 2\theta=-\frac{8}{6}\Rightarrow\cos 2\theta=-\frac{4}{5}\Rightarrow\cos\theta=\frac{1}{\sqrt{10}},\ \sin\theta=\frac{3}{\sqrt{10}}$$

c) $\frac{1}{10}\left[(x'-3y')^2+6(x'-3y')(3x'+y')+9(3x'+y')^2\right]+\frac{1}{\sqrt{10}}\sqrt{10}(x'-3y')=18$

$$\frac{1}{10}\left[x'^2(1+18+81)+x'y'(-6+6-54+54)+y'^2(9-18+9)\right]+x'-3y'=18$$

$$\frac{1}{10}\left(100x'^2\right)+x'=3y'+18=10x'^2+x\Rightarrow10\left(x'^2+.05\right)^2=3(y'+6.025)$$

5. Accurately graph these conics which attention to the usual (starting with eccentricity)

$a)\ r=\dfrac{30}{2-8\cos\theta}=\dfrac{15}{1-4\cos\theta}$, $e=4$, *hyperbola*, $p=\dfrac{15}{4}$, *vertices* : $(-3,0),(-5,0)$,

center : $(-4,0)$, *directrices* : $x=-4\pm\dfrac{1}{4}$, *foci* : $(0,0)$, $(-8,0)$, *as* : $y=\pm\sqrt{15}(x+4)$

$LR=30$

$b)\ r=\dfrac{30}{8+2\sin\theta}=\dfrac{\frac{15}{4}}{1+\frac{1}{4}\sin\theta}$, $e=\dfrac{1}{4}$, *ellipse*, $p=15$, *vertices* $(0,3),(0,-5,)$, *center* $(0,-1,)$

directrices : $x=15$, $x=-17$, *foci* $(0,0),(-2.0)$, minor *axis* : $2\sqrt{15}$

$c)\ r=\dfrac{10}{3+3\cos\theta}=\dfrac{\frac{10}{3}}{1+\cos\theta}$, $e=1$, *parabola*, $p=\dfrac{10}{3}$, *vertex* $\left(\dfrac{5}{3},0\right)$, *focus* $(0,0)$

directrix $x=\dfrac{10}{3}$, $LA=\dfrac{20}{3}$

6. Convert to rectangular form:

$a)\ \begin{array}{c} x=3\cos t \\ y=-5\sin t \end{array}$ $\quad b)\ \begin{array}{c} x=\cos 2t \\ y=\sin t \end{array}$ $\quad c)\ \begin{array}{c} x=e^{3t} \\ y=e^{-5t} \end{array}$ $\quad d)\ r=\tan\theta$

7. An searchlight mirror in the form of a parabola of revolution has focal length of 3’. Its maximum diameter is 4’. What is the depth of the mirror.

$x^2=4py=2^2=4\bullet 3y$

$y=\dfrac{1}{3}$ *foot*