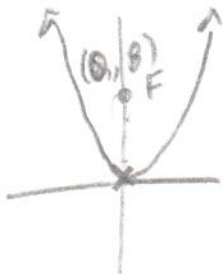


Conic Review Sheet

Stand up conic Key

1. A hyperbola and parabola share a focus along the positive y axis. The vertex of the parabola is located at the second focus of the hyperbola. The equation of the parabola is  $x^2 = 32y$ . A vertex of the hyperbola is at  $(0, 7)$ . Find the equation of the hyperbola.



$$4a = 32$$

$$a = 8$$

$$\frac{(y-4)^2}{9} - \frac{x^2}{7} = 1$$

$$a = 3 \quad c^2 = a^2 + b^2$$

$$c = 4 \quad 16 = 9 + b^2$$

2. Given the polar equation  $r = \frac{2}{3 + 5\cos\theta}$ . What kind of conic is it? What is e? What is the equation of the directrix? What is the rectangular form?

$$\frac{2/3}{1 + 5/3 \cos\theta}$$

$$e = 5/3$$

$$h = 2/5$$

$$r \cos\theta = 2/5$$

↑  
directrix

$$3r + 5r \cos\theta = 2$$

$$3\sqrt{x^2 + y^2} = 2 - 5x$$

$$9x^2 + 9y^2 = 4 - 20x + 25x^2$$

$$16x^2 - 20x - 9y^2 = -4$$

3. Rotate the axes by 30 degrees to determine the new conic:  $x^2 - \sqrt{3}xy + 2y^2 = 2$

$$\tan 2\theta = \frac{-\sqrt{3}}{1-2} = \sqrt{3}$$

$$\theta = 30^\circ$$

$$x = x' \frac{\sqrt{3}}{2} - y' \frac{1}{2}$$

$$y = x' \frac{1}{2} + y' \frac{\sqrt{3}}{2}$$

$$\left(x \frac{\sqrt{3}}{2} - y \frac{1}{2}\right)^2 - \sqrt{3} \left(x \frac{\sqrt{3}}{2} - y \frac{1}{2}\right) \left(x \frac{1}{2} + y \frac{\sqrt{3}}{2}\right) + 2 \left(x \frac{1}{2} + y \frac{\sqrt{3}}{2}\right)^2 = 2$$

$$\frac{3}{4}x^2 - \frac{2 \times 4 \sqrt{3}}{4}xy + \frac{1}{4}y^2 - \sqrt{3} \left[ x^2 \frac{\sqrt{3}}{4} + xy \frac{3}{4} - xy \frac{1}{4} - \frac{\sqrt{3}}{4}y^2 \right] + 2 \left[ \frac{x^2}{4} + \frac{2\sqrt{3}}{4}xy + \frac{3}{4}y^2 \right] = 2$$

$$\frac{x^2}{2} + \frac{5}{2}y^2 = 2$$

$$\frac{x^2}{4} + \frac{5}{4}y^2 = 1 \quad \text{ellipse}$$

4. Given the degenerate conic  $x^2 - xy - 2y^2 + x - 2y = 0$ . What kind of degenerate is it? What kind of degenerate are you? Change the form of the degenerate into a more "proper" form corresponding with the shape of the curve.

$$1 - 4 \cdot 1 \cdot -2 = 9$$

$$1 + 8 = 9$$

$$B^2 - 4AC > 1$$

Ellipse

$$2y^2 + xy + 2y + (-x^2 - x) = 0$$

$$\frac{-(2+x) \pm \sqrt{(2+x)^2 + 4 \cdot 2 \cdot (x^2+x)}}{4}$$

$$= \frac{-(2+x) \pm \sqrt{4 + 4x + x^2 + 8x^2 + 8x}}{4}$$

$$= \frac{-(2+x) \pm \sqrt{9x^2 + 12x + 4}}{4}$$

$$y = \frac{-(2+x) \pm \sqrt{(3x+2)^2}}{4}$$

$$= \frac{-(2+x) + 3x+2}{4}$$

$$y = \frac{2x}{4} \quad \boxed{y = x/2}$$

$$y = \frac{-2-x-3x-2}{4} \quad \boxed{y = -x-1}$$

$$\frac{10}{16} = \frac{5}{8}$$

$$16 \left( x^2 - \frac{20}{16}x \right) - 9y^2 = -4$$

$$16 \left( x^2 - \frac{5}{4}x + \frac{25}{64} \right) - 9y^2 = -4 + 16 \left( \frac{25}{64} \right)$$

$$\frac{(x - \frac{5}{8})^2}{\frac{9}{64}} - \frac{y^2}{\frac{9}{36}} = 1$$

$$-\frac{16}{4} + \frac{25}{4} = \frac{9}{4}$$