$\frac{24}{16}$ $\frac{5}{4}$

$16\left(x^2 - \frac{5}{4}x + \frac{25}{64}\right) - 9y^2 = -4$ $-4 + \frac{25}{64}$

$\frac{25}{64}$

Conic Review Sheet

Stand up conic Key

1. A hyperbola and parabola share a focus along the positive y axis. The vertex of the parabola is located at the second focus of the hyperbola. The equation of the parabola is $x^2 = 32y$. A vertex of the hyperbola is at (0, 7). Find the equation of the hyperbola.

$4a = 32$

$a = 8$

$$\frac{(y-4)^2}{9} - \frac{x^2}{7} = 1$$

$a = 3$ $c^2 = a^2 + b^2$

$c = 4$ $16 = 9 + b^2$

2. Given the polar equation $r = \dfrac{2}{3 + 5\cos\theta}$. What kind of conic is it? What is e? What is the equation of the directrix? What is the rectangular form?

hyperbola $5/2$

$3r + 5r\cos\theta = 2$

$\dfrac{2/3}{1 + \frac{5}{3}\cos\theta}$ $e = 5/3$ $h = 2/5$

$h \cdot e = 2/3$

$h \cdot 5/3 = 2/3$

$h = (2/3)(3/5)$

$\boxed{r\cos\theta = 2/5}$

directrix

$3\sqrt{x^2+y^2} = 2 - 5x$

$9x^2 + 9y^2 = 4 - 20x + 25x^2$

$16x^2 - 20x - 9y^2 = -4$

3. Rotate the axes by 30 degrees to determine the new conic: $x^2 - \sqrt{3}xy + 2y^2 = 2$

$\tan 2\theta = \dfrac{-\sqrt{3}}{1-2} = \sqrt{3}$

$\theta = 30°$

$x = x'\frac{\sqrt{3}}{2} - y'\frac{1}{2}$

$y = x' \cdot \frac{1}{2} + y'\frac{\sqrt{3}}{2}$

$\left(x \cdot \frac{\sqrt{3}}{2} - y \cdot \frac{1}{2}\right)^2 - \sqrt{3}\left(x'\frac{\sqrt{3}}{2} - y' \cdot \frac{1}{2}\right)\left(x' \cdot \frac{1}{2} + y'\frac{\sqrt{3}}{2}\right) + 2\left(x' \cdot \frac{1}{2} + y'\frac{\sqrt{3}}{2}\right)^2$

$\frac{3}{4}x^2 - \frac{2xy\sqrt{3}}{4} + \frac{1}{4}y^2 - \sqrt{3}\left[x^2 \cdot \frac{\sqrt{3}}{4} + xy\frac{3}{4} - xy \cdot \frac{1}{4} - \frac{\sqrt{3}}{4}y^2\right] + 2\left[\frac{x^2}{4} + \frac{2\sqrt{3}}{4}xy + \frac{3}{4}y^2\right]$

$xy/2$

$\cancel{\frac{3}{4}x^2} - \cancel{\frac{\sqrt{3}}{2}xy} + \frac{1}{4}y^2 - \cancel{\frac{3}{4}x^2} - \cancel{\frac{\sqrt{3}}{2}xy} + \frac{3}{4}y^2 + \frac{x^2}{2} + \cancel{\sqrt{3}xy} + \frac{3}{2}y^2$

$\frac{x^2}{2} + \frac{5}{2}y^2 = 2$ $\boxed{\frac{x^2}{4} + \frac{5y^2}{4} = 1}$ ellipse

4. Given the degenerate conic $x^2 - xy - 2y^2 + x - 2y = 0$. What kind of degenerate is it? What kind of degenerate are you? Change the form of the degenerate into a more "proper" form corresponding with the shape of the curve.

$1 - 4 \cdot 1 \cdot -2 =$

$1 + 8 = 9$

$B^2 - 4AC > 1$

Ellipse

$2y^2 + \underbrace{xy + 2y}_{(2+x)y} + (-x^2 - x) = 0$

$\dfrac{-(2+x) \pm \sqrt{(2+x)^2 + 4 \cdot 2 \cdot (x^2 + x)}}{4}$

$= \dfrac{-(2+x) \pm \sqrt{4 + 4x + x^2 + 8x^2 + 8x}}{4}$

$= \dfrac{-(2+x) \pm \sqrt{9x^2 + 12x + 4}}{4}$

$y = \dfrac{-(2+x) \pm \sqrt{(3x+2)^2}}{4}$

$= \dfrac{-(2+x) + 3x + 2}{4}$

$y = \dfrac{2x}{4}$ $\boxed{y = x/2}$

$y = \dfrac{-2 - x - 3x - 2}{4}$ $\boxed{y = -x - 1}$

$\frac{10}{16} = \frac{5}{8}$ $\qquad 16\left(x^2 - \frac{20}{16}x \qquad\right) - 9y^2 = -4$

$$16\left(x^2 - \frac{5}{\cancel{4}}x + \frac{25}{64}\right) - 9y^2 = -4 + 16\left(\frac{25}{64}\right)$$

$$\frac{(x - 5/8)^2}{9/64} - \frac{y^2}{9/36} = 1 \qquad -\frac{16}{4} + \frac{25}{4} = \frac{9}{4}$$